RLC SUMMARY

I. Summary of RLC

A. Initial/ final values – short L, open C and: $v_C(0^+) = v_C(0^-)$ $i_L(0^+) = i_L(0^-)$

1. For derivatives: $\dot{v}, i'$ use $i_C = C\dot{v}$ and $v_L = Li'$

II. SOURCE FREE

Σ RLC

$i'' + (R/L)i' + (1/LC)i = 0$ (KVL)

(alt) $\ddot{v}_C + (R/L)\dot{v}_C + (1/LC)v_C = 0$

// RLC

$\ddot{v} + (1/RC)\dot{v} + (1/LC)v = 0$

(alt) $i_L'' + (1/RC)i_L' + (1/LC)i_L = 0$

$$s^2 + 2\alpha s + \omega_O{}^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_O{}^2}$$

$\alpha = R/(2L)$; $\omega_O = 1/\sqrt{LC}$ $\alpha = 1/(2RC)$; $\omega_O = 1/\sqrt{LC}$

Solutions

$\alpha > \omega_O$ (overd) $s_{1,2}$ real, (–), distinct

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$\alpha = \omega_O$ (crit) $s_{1,2}$ real, (–), equal

$i(t) = e^{-\alpha t}(A_2 + A_1 t)$ $v(t) = e^{-\alpha t}(A_2 + A_1 t)$

$\alpha < \omega_O$ (underd) $s_{1,2}$ complex, distinct

$i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$ $v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

$\omega_d = \sqrt{\omega_O{}^2 - \alpha^2}$

BCs

$i(0) = I_O$

$i'(0) = -(RI_0 + V_0)/L$

BCs

Vo

$\dot{v}(0) = -(V_O + I_O R)/(RC)$

OR $i'(0) = v_O / L$ (for i eqn)

STEP RESPONSE

Σ RLC (step w/ VS), $v = v_C$

$\ddot{v} + (R/L)\dot{v} + (1/LC)v = V_S(1/LC)$

(alt) $i'' + (R/L)i' + (1/LC)i = 0$

// RLC (step with CS, $i = i_L$)

$i_L'' + (1/RC)i' + (1/LC)i = I_S(1/LC)$

(alt) $\ddot{v} + (1/RC)\dot{v} + (1/LC)v = 0$

Step Solns

$v_C(t) = V_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$v(t) = V_S + (A_1 + A_2 t)e^{-\alpha t}$

$v(t) = V_S + e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

$\alpha = R/(2L)$; $\omega_O = 1/\sqrt{LC}$

Step Solns

$i(t) = I_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$i(t) = I_S + (A_1 + A_2 t)e^{-\alpha t}$

$i(t) = I_S + e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

$\alpha = 1/(2RC)$; $\omega_O = 1/\sqrt{LC}$

Diff Eq's are now NON-homogeneous. Solutions have a transient (homogeneous) and steady-state component (particular). The transient solution is associated with the source-free response. The steady-state is associated with the "driving" element (VS or CS).

$v(t) = v_{ss} + v_t$ $\quad$ $i(t) = i_{ss} + i_t$

$v(t) = V_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\quad$ $i(t) = I_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (over)

$v(t) = V_S + e^{-\alpha t}(A_1 + A_2 t)$ $\quad$ $i(t) = I_S + e^{-\alpha t}(A_1 + A_2 t)$ (crit)

$v(t) = V_S + e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$ $\quad$ $i(t) = I_S + e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$ (under)

III. Solving problems

A. Get initial values: $i(0^-)$, $v(0^-)$ (short L, open C, do analysis as usual)

B. Get values t = 0+ (just after event).
   1. $i_L(0^+) = i_L(0^+)$ (i on L contin.)
   2. $v_C(0^+) = v_C(0^-)$ (v on C contin)
   3. $i'(0^+) \leftarrow i'(0^+) = v_L(0^+)/L$ (get via voltage across L)
   4. $\dot{v}(0^+) \leftarrow \dot{v}(0^+) = i_C(0^+)/C$ (get via current thru C)

C. Get transient response
   1. Get α & ω_O (depends on circuit, series, parallel, etc.)
   2. Is it over, under, or critically damped?
   3. Compute roots s_1 & s_2.
   4. Pick proper equation (based on damping)
   5. Get A_1 & A_2 from BC's.
      a. For source-free series RLC we want $i(0+), i'(0+)$
      b. For source-free parallel RLC we want: $v(0+), \dot{v}(0+)$
      c. For step-response series RLC we want $v(0+), \dot{v}(0+)$
      d. For step-response parallel RLC we want $i(0+), i'(0+)$

D. Get final response (t = ∞) – just like t < 0
   1. Note – steady state response of source-free is always = 0 (no source, decays to 0)
   2. Step response variable is opposite to source-free (v for series RLC, I for // RLC) (it STEPS)

IV. Solving GENERAL second order circuits (SKIP FOR NOW)

A. Get initial conditions: $x(0)$, $\dot{x}(0)$, and final conditions $x(\infty)$

B. Get transient response $x_t(t)$
   1. Turn off independent sources
   2. Use KCL or KVL.
   3. Get s1, s2
   4. Get damping type (under, critically, over damped).
   5. Get A_1 & A_2 from BCs.

C. Get steady state response $x_{ss} = x(\infty)$

D. Get total response $x(t) = x_t(t) + x_{ss}(t)$