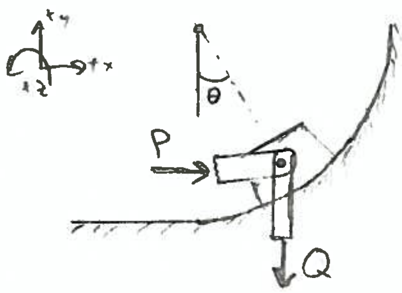
CH 8

VECTOR MECHANICS FOR ENGINEERS - STATICS - 9TH EDITION.

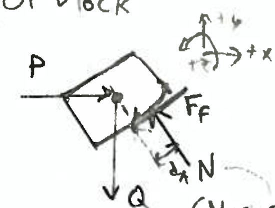
8.6) - A block on a circular incline.

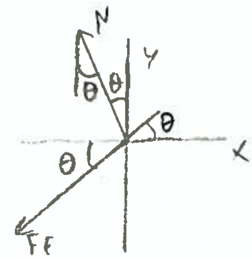
Given: $\mu_s = 0.25$
$\mu_k = 0.20$
$\theta = 30°$
$Q = 500\ N$

Find: The range of values of P for which equilibrium is maintained.



FBD for (A) of Block



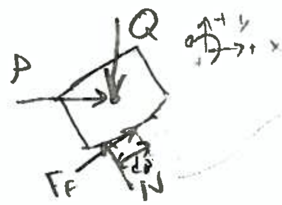


(N is offset in order to counteract the moment produced by F_f)

We have two scenarios:

* P is large and motion is impending up the incline — Situation (A)
* P is small and motion is impending down the incline — Situation (B)

FBD for (B) of block



For both situations, F_f is maximized, meaning $F_f = \mu_s N$

So we have

(A) $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow P_{max} - N\sin\theta - \underbrace{F_f}_{\mu_s N}\cos\theta = 0$ (A)$_x$

$+\uparrow \Sigma F_y = 0 \Rightarrow -Q + N\cos\theta - \underbrace{F_f}_{\mu_s N}\sin\theta = 0$, $N(\cos\theta - \mu_s\sin\theta) = Q$

$N = \dfrac{Q}{\cos\theta - \mu_s\sin\theta}$

Plugging into (A)$_x$

$P_{max} - N\sin\theta - \mu_s N\cos\theta = 0$, $P_{max} = N(\sin\theta + \mu_s\cos\theta)$

$P_{max} = \dfrac{Q(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}$, $P_{max} = \dfrac{500\ N(\sin 30° + (0.25)\cos 30°)}{\cos 30° - 0.25\sin 30°}$

$P_{max} = 483\ N$

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8.6 Part II)

And for B, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow P_{min} - N\sin\theta + \underbrace{F_f}_{\mu_s N}\cos\theta = 0, \quad P_{min} = N(\sin\theta - \mu_s\cos\theta)$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow -Q + \underbrace{F_f}_{\mu_s N}\sin\theta + N\cos\theta = 0.$$

$$N(\mu_s \sin\theta + \cos\theta) = Q$$

$$N = \frac{Q}{\mu_s \sin\theta + \cos\theta}$$

So, plugging N into P_{min}:

$$P_{min} = \left(\frac{Q}{\mu_s \sin\theta + \cos\theta}\right)(\sin\theta - \mu_s\cos\theta)$$

$$P_{min} = \left(\frac{500N}{(0.25)\sin 30^\circ + \cos 30^\circ}\right)(\sin 30^\circ - (0.25)\cos 30^\circ)$$

$$P_{min} = 143.0\,N$$

Therefore, the range of values for P is:

$$P_{min} \leq P \leq P_{max}$$

$$\boxed{143.0N \leq P \leq 483N}$$

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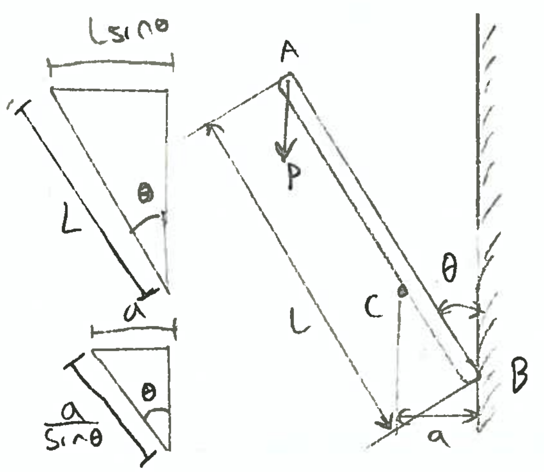
8.29) A Slender rod lodged between a pin and a wall

Given: L $\theta = 35^\circ$

P

$\mu_s = 0.20$ @ B and C

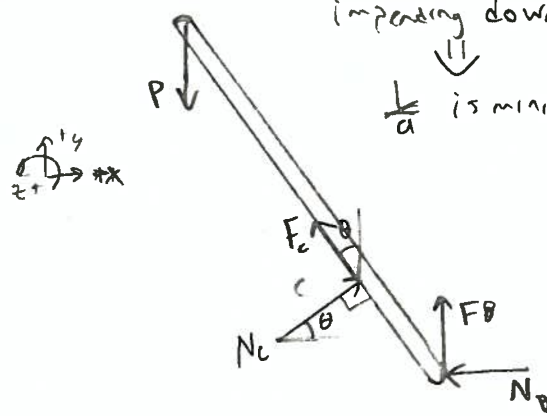
Find: $\frac{L}{a}$ values for which equilibrium is maintained



(A) FBD of rod (motion of end B impending downward)

⇓

$\frac{L}{a}$ is minimized



From (A):

$+\rightarrow \Sigma F_x = 0 \Rightarrow N_c \cos\theta - \underset{\mu_s N_c}{F_c} \sin\theta - N_B = 0$

eqn ① $N_B = N_c \cos\theta - \mu_s N_c \sin\theta$

$+\uparrow \Sigma F_y = 0 \Rightarrow -P + N_c \sin\theta + \underbrace{F_c}_{\mu_s N_c} \cos\theta + \underbrace{F_B}_{\mu_s N_B} = 0$

eqn ② $-P + N_c \sin\theta + \mu_s N_c \cos\theta + \mu_s N_B = 0$

① → ② ⇓

$-P + N_c \sin\theta + \mu_s N_c \cos\theta + \mu_s(N_c \cos\theta - \mu_s N_c \sin\theta) = 0$

$N_c(\sin\theta + \mu_s \cos\theta + \mu_s \cos\theta - \mu_s^2 \sin\theta) = P$

$$N_c = \frac{P}{\sin\theta + 2\mu_s \cos\theta - \mu_s^2 \sin\theta} \quad ②^*$$

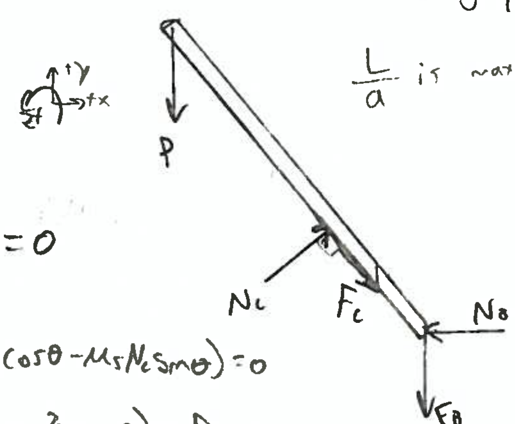
$+\circlearrowleft \Sigma M_B = 0 \Rightarrow P L \sin\theta - N_c \frac{a}{\sin\theta} = 0$

$P L \sin\theta = N_c \frac{a}{\sin\theta}$

eqn ③ $\frac{L}{a} = \frac{N_c}{P \sin^2\theta}$

(B) FBD of rod (motion of end B impending upward)

$\frac{L}{a}$ is maximized



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8.29 Part II)

② → ③

$\Downarrow$

$$\frac{L}{a}_{min} = \frac{\dfrac{\cancel{P}}{\sin\theta + 2\mu_s\cos\theta - \mu_s^2\sin\theta}}{\dfrac{\cancel{P}\sin^2\theta}{1}}, \quad \frac{L}{a} = \frac{1}{\sin^2\theta(\sin\theta + 2\mu_s\cos\theta - \mu_s^2\sin\theta)}$$

Because in Ⓐ $\frac{L}{a}$ is being minimized.

$$\frac{L}{a}_{min} = \frac{1}{\sin^2(35°)(\sin 35° + 2(0.20)\cos 35° - (0.20)^2\sin 35°)}$$

$$\frac{L}{a}_{min} = 3.46$$

From Ⓑ:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow N_c\cos\theta + F_c\sin\theta - N_B = 0$$

$$\text{eqn ④} \quad N_B = N_c\cos\theta + \mu_s N_c\sin\theta$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow -P + N_c\sin\theta - F_c\cos\theta - F_B = 0$$

$$\text{eqn ⑤} \quad -P + N_c\sin\theta - N_c\mu_s\cos\theta - N_B\mu_s = 0$$

④ → ⑤

$$-P + N_c\sin\theta - N_c\mu_s\cos\theta - (N_c\cos\theta + \mu_s N_c\sin\theta)\mu_s = 0$$

$$N_c(\sin\theta - \mu_s\cos\theta - \mu_s\cos\theta - \mu_s^2\sin\theta) = P$$

$$N_c = \frac{P}{\sin\theta - 2\mu_s\cos\theta - \mu_s^2\sin\theta}$$

$$\circlearrowleft + \Sigma M_B = 0 \Rightarrow PL\sin\theta - N_c\frac{a}{\sin\theta} = 0$$

which gives $③_{max} \Rightarrow \quad \frac{L}{a} = \frac{N_c}{P\sin^2\theta}$

So,

$$\frac{L}{a}_{max} = \frac{1}{\sin^2\theta(\sin\theta - 2\mu_s\cos\theta - \mu_s^2\sin\theta)}$$

$$\frac{L}{a}_{max} = \frac{1}{\sin^2(35)(\sin 35° - 2(0.20)\cos 35° - (0.20)^2\sin 35°)}, \quad \frac{L}{a}_{max} = 13.63$$

$\therefore$ The range of values of $\frac{L}{a}$ for which equilibrium is maintained is: $\boxed{3.46 \le \frac{L}{a} \le 13.63}$