CH 7

VECTOR MECHANICS FOR ENGINEERS - STATICS - 9TH EDITION

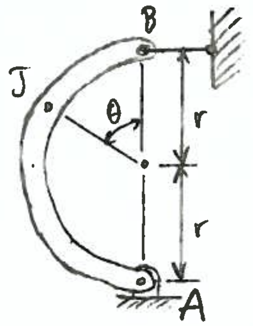
7.23) A Semicircular rod

Given: · W (weight of rod)

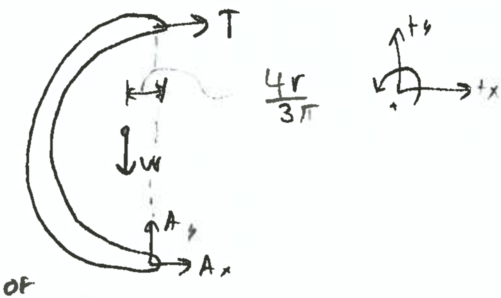
· r

· θ: 60°

Find: the Bending moment at point J



FBD of AB

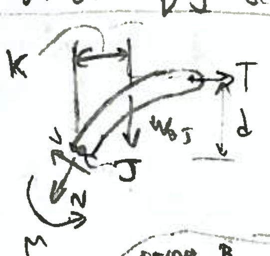


from back of book table of centroids

$$\circlearrowleft + \; \Sigma M_A = 0 \Rightarrow \quad W\left(\frac{2r}{\pi}\right) - T(2r) = 0$$

$$T = \frac{W}{\pi}$$

FBD of BJ section of semicircular rod:

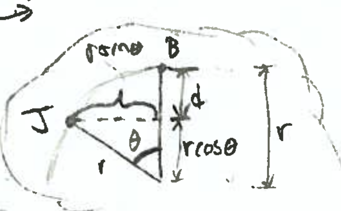


eqn *

$$\circlearrowleft + \; \Sigma M_J = 0 \Rightarrow \; -Td - W_{BJ}k + M = 0$$

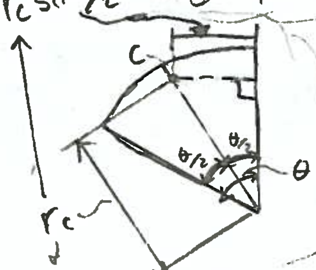
So from the figures, we see that:

$$\underline{d = r - r\cos\theta}$$



$d = r - r\cos\theta$

$r_c\sin\theta/2$



$$r_c = \frac{r\sin\left(\frac{\theta}{2}\right)}{\frac{\theta}{2}}$$

→ From back of book Table of Centroids

$$k = r\sin\theta - r_c\sin\left(\frac{\theta}{2}\right)$$

$$= r\sin\theta - \left(\frac{2r\sin\left(\frac{\theta}{2}\right)}{\theta}\right)\sin\left(\frac{\theta}{2}\right)$$

$$\underline{k = r\left(\sin\theta - \frac{2\sin^2\left(\frac{\theta}{2}\right)}{\theta}\right)}$$

Also, the weight W_{BJ} is a proportion:

$$\frac{(\text{total weight})(\text{length of section of arc})}{(\text{total arc length})} = \text{weight of section}$$

Or $\quad \dfrac{W(\theta \cancel{r})}{\pi \cancel{r}} = W_{BJ}$, $\quad \underline{W_{BJ} = \dfrac{W\theta}{\pi}}$

and we found that:

$$\underline{T = \frac{W}{\pi}}$$

7.23 Part II)

So, now we have from eqn $*$:

$$-\left(\frac{W}{\pi}\right)(r - r\cos\theta) - \left(\frac{W\theta}{\pi}\right)\left[r\left(\sin\theta - \frac{2\sin^2\left(\frac{\theta}{2}\right)}{\theta}\right)\right] + M = 0$$

$$M = \left(\frac{W}{\pi}\right)(r - r\cos\theta) - \left(\frac{W\theta}{\pi}\right)\left[r\sin\theta - \frac{2r\sin^2\left(\frac{\theta}{2}\right)}{\theta}\right]$$

$$M = \frac{Wr}{\pi}\left[1 - \cos\theta + \theta\sin\theta - 2\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$M = \frac{Wr}{\pi}\left[1 - \cos(60^\circ) + (60^\circ)\left(\frac{\pi}{180^\circ}\right)\sin(60^\circ) - (2)\left(\sin^2\left(\frac{60^\circ}{2}\right)\right)\right]$$

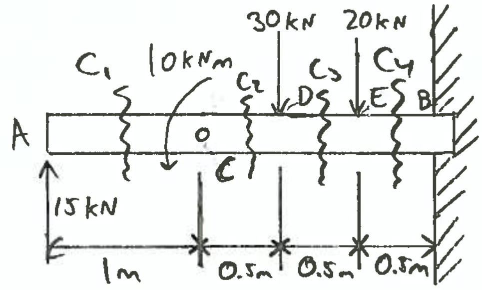
$$\boxed{M = 0.289\,Wr \circlearrowleft}$$

Ch 7

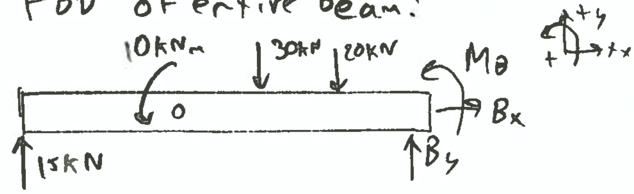
7.35) A loaded cantilevered beam

Given: the Following diagram

Find: (a) V & M (Shear & Moment) Diagrams
(b) $|V_{max}|$ & $|M_{max}|$



FBD of entire beam:



$+\rightarrow \Sigma F_x = 0 \Rightarrow B_x = 0$

$+\uparrow \Sigma F_y = 0 \Rightarrow B_y + 15kN - 30kN - 20kN = 0$

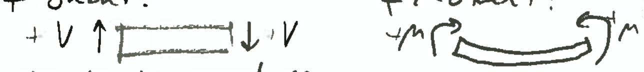
$B_y = 35\,kN \uparrow$

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow M_B + (20kN)(0.5m) + (30kN)(1m) + 10kNm - (15kN)(2.5m) = 0$

$M_B = -12.5\,kNm$ or $M_B = 12.5\,kNm \circlearrowright$

Note on Sign Convention:

+ Shear:

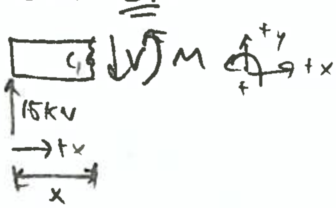


Tends to produce Clockwise rotation

+ moment:

Tends to make a "Happy Face"

FBD of left side of cut C_1:



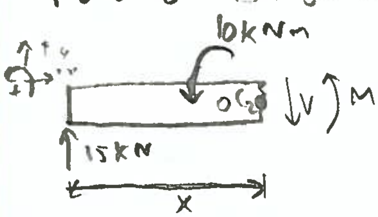
$+\uparrow \Sigma F_y = 0 \Rightarrow 15kN - V = 0$

$V = 15kN$

$+\circlearrowleft \Sigma M_{C_1} = 0 \Rightarrow M - 15kN\,x = 0$

$M = 15x$ → Applicable Only between A & C

FBD of left side of cut C_2

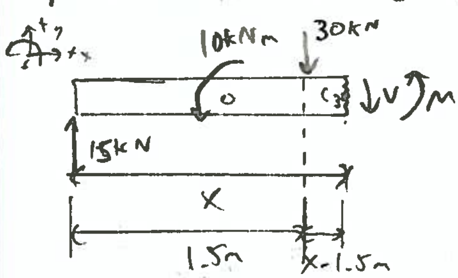


$+\uparrow \Sigma F_y = 0 \Rightarrow 15kN - V = 0$, $V = 15kN$

$+\circlearrowleft \Sigma M_{C_2} = 0 \Rightarrow M + 10kNm - (15kN)x = 0$

$M = 15x - 10$

FBD of left side of cut C_3:



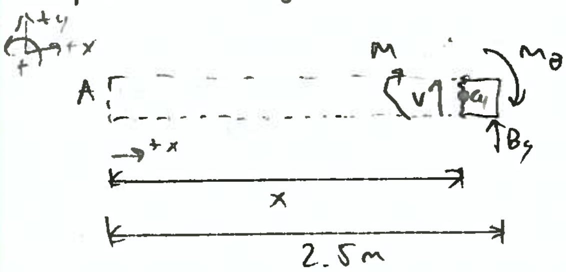
$+\uparrow \Sigma F_y = 0 \Rightarrow 15kN - 30kN - V = 0$, $V = -15kN$

$+\circlearrowleft \Sigma M_{C_3} = 0 \Rightarrow M + 10kNm - 15kNx + (30kN)(x - 1.5m) = 0$

$M = -15x + 35$

Ch7
7.35 Part II)

FBD of right side of cut C_4:



$+\uparrow \sum F_y = 0 \Rightarrow V + B_y = 0$
$V = -B_y$, $V = -35\,kN$

$\circlearrowleft + \sum M_{C4} = 0 \Rightarrow -M - M_o + B_y(2.5m - x) = 0$

$M = -M_B + B_y(2.5 - x)$

$M = -(12.5\,kNm) + (35\,kN)(2.5 - x)$

$M = -35x + 75$

Using the eqns obtained from each cut we can now evaluate the V & M values @ A, B, C, D & E to draw our V & M diagrams.

A: ($x = 0m$)
Using C_1 eqns
$V(0m) = 15\,kN$
$M(0m) = 15(0) = 0\,kNm$

C: ($x = 1m$)
using C_1 eqns:
$V(1m) = 15\,kN$
$M(1m) = 15\,kNm$

Using C_2 eqns
$V(1m) = 15\,kN$
$M(1m) = 15\,kN(1m) - 10\,kNm = 5\,kNm$

D: ($x = 1.5m$)
using C_2 eqns:
$V(1.5m) = 15\,kN$
$M(1.5m) = 15(1.5) - 10$
$= 12.5\,kNm$

using C_3 eqns:
$V(1.5m) = -15\,kN$
$M(1.5m) = -15(1.5m) + 35$
$= 12.5\,kNm$

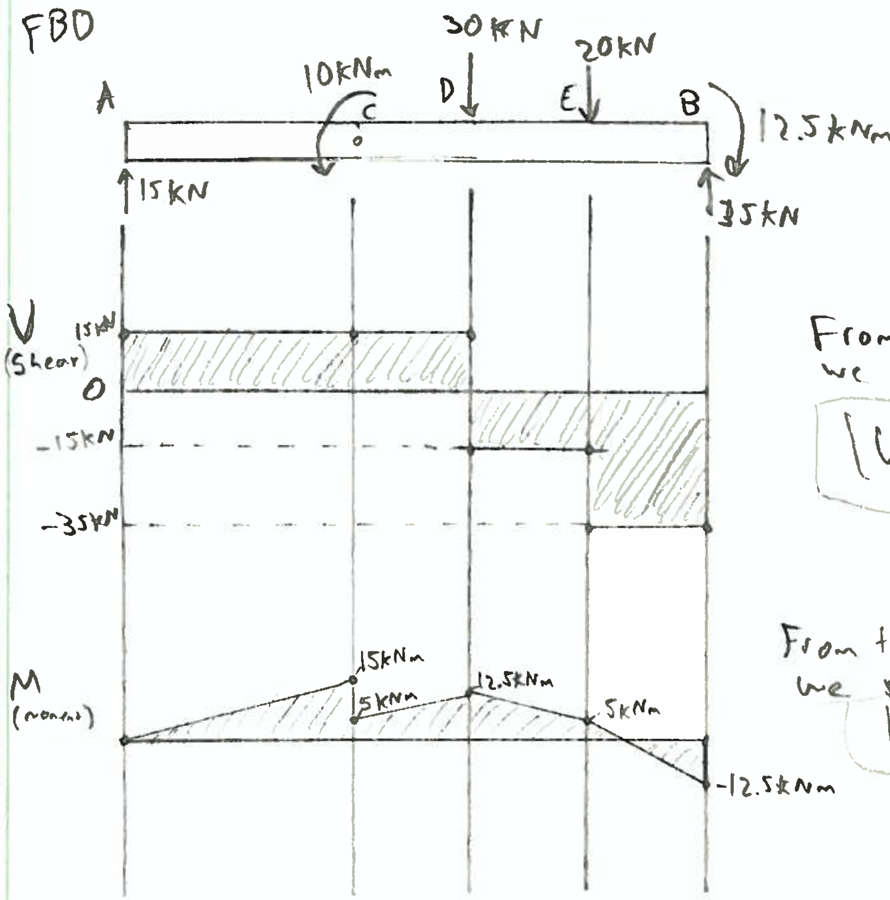
E: ($x = 2m$)
using C_3 eqns:
$V(2m) = -15\,kN$
$M(2m) = -15(2) + 35$
$= 5\,kNm$

using C_4 eqns
$V(2m) = -35\,kN$
$M(2m) = -35(2) + 75$
$= 5\,kNm$

B:
using C_4 eqns:
$V(2.5m) = -35\,kN$
$M(2.5m) = -35(2.5) + 75 = -12.5\,kNm$

7.35 Part III)

Now we can draw our V & M diagrams



From the shear diagram, we see that

$$\boxed{|V_{max}| = 35\,kN}$$

From the moment diagram, we see that:

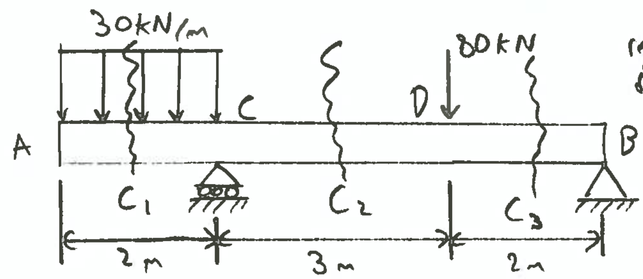
$$\boxed{|M_{max}| = 15\,kNm}$$

Ch7

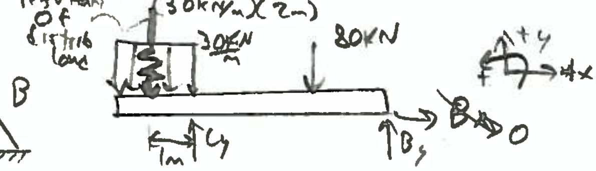
7.40) A loaded overhanging beam

Given: the following diagram

Find: (a) the V & M diagrams
(b) $|V_{max}|$ & $|M_{max}|$



FBD of entire Beam



$\circlearrowleft + \; \Sigma M_B = 0 \Rightarrow (80kN)(2m) - C_y(5m) + [(30kN/m)(2m)](6m) = 0$

$C_y = \dfrac{(80kN)(2m) + (30kN/m)(2m)(6m)}{5m}$

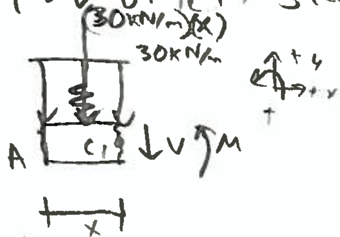
$C_y = 104kN \uparrow$

$+\uparrow \Sigma F_y = 0 \Rightarrow -(30kN/m)(2m) - 80kN + C_y + B_y = 0$

$B_y = 60kN + 80kN - 104kN$

$B_y = 36kN \uparrow$

FBD of left side of cut C_1



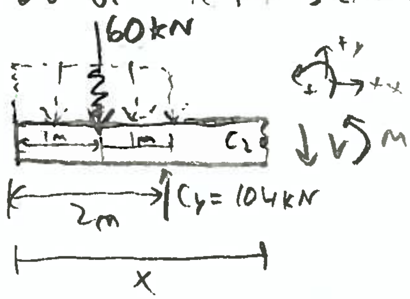
$+\uparrow \Sigma F_y = 0 \Rightarrow -30x - V = 0$

$V = -30x$

$\circlearrowleft + \; \Sigma M_{C_1} = 0 \Rightarrow M + (30x)\left(\frac{x}{2}\right) = 0$

$M = -15x^2$

FBD of left side of cut C_2:



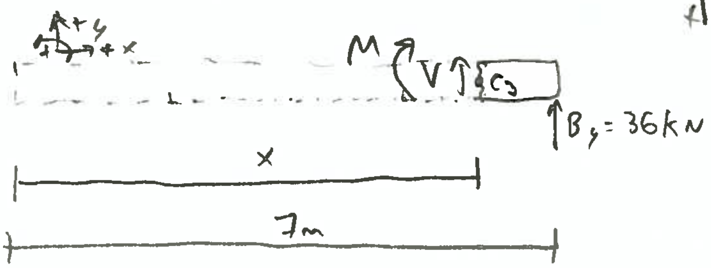
$+\uparrow \Sigma F_y = 0 \Rightarrow -60kN + 104kN - V = 0$

$V = 44kN$

$\circlearrowleft + \; \Sigma M_{C_2} = 0 \Rightarrow (60kN)(x - 1m) - 104kN(x - 2m) + M = 0$

$M = 44x - 148$

FBD of right side of cut C_3.



$+\uparrow \Sigma F_y = 0 \Rightarrow V + 36kN = 0$

$V = -36kN$

$\circlearrowleft + \; \Sigma M_{C_3} = 0 \Rightarrow -M + 36kN(7 - x) = 0$

$M = -36x + 252$

Ch7
7.40 Part II)

Evaluating at points A, B, C & D:

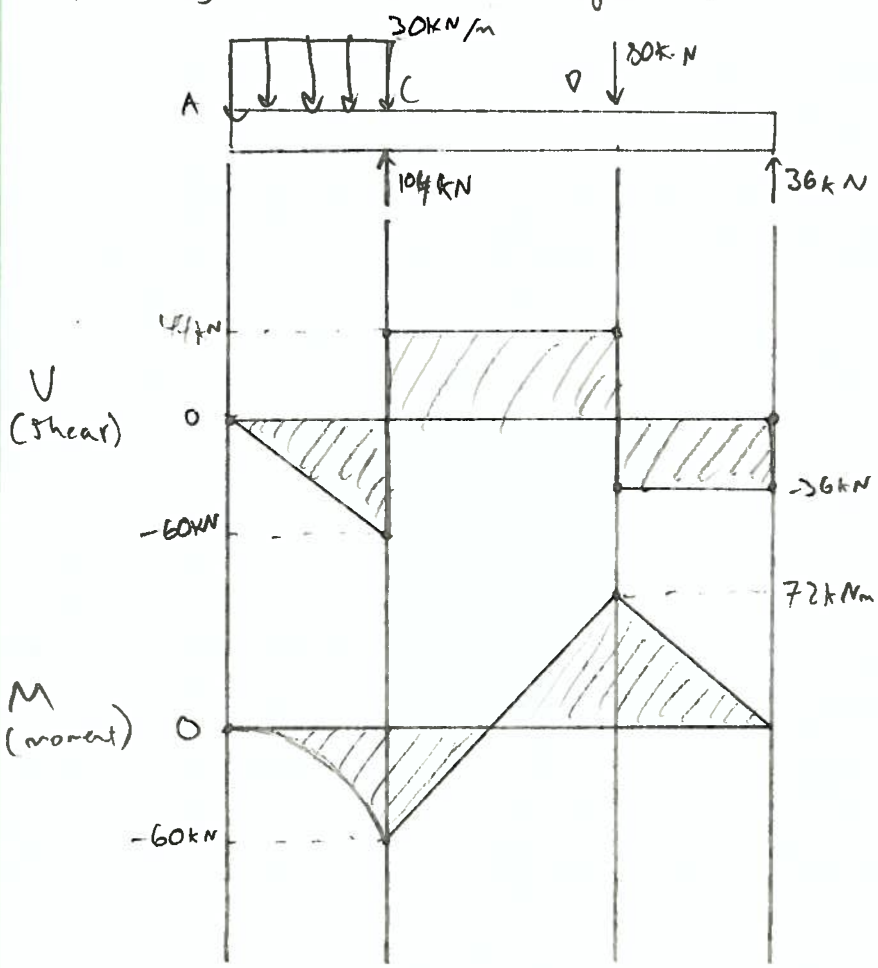
A: (x=0) using C_1: $V(0m) = 0$ kN
$M(0m) = 0$ kNm

C: (x=2m) using C_1: $V(2m) = -60$ kN
$M(2m) = -60$ kNm
using C_2: $V(2m) = 44$ kN
$M(2m) = -60$ kNm

D: (x=5m) using C_2: $V(5m) = 44$ kN
$M(5m) = 72$ kNm
using C_3: $V(5m) = -36$ kN
$M(5m) = 72$ kNm

B: (x=7m) using C_3:
$V(7m) = -36$ kN
$M(7m) = 0$

Drawing V & M diagrams:



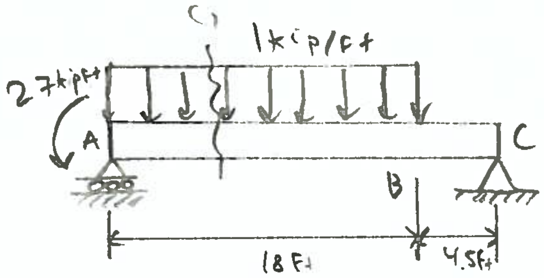
So, $|V_{max}| = 60$ kN & $|M_{max}| = 72$ kNm

Ch 7

7.79) A loaded simply supported beam

Given: The following diagram

Find: (a) Draw the V & M diagrams
(b) magnitude and location of M_{max}.



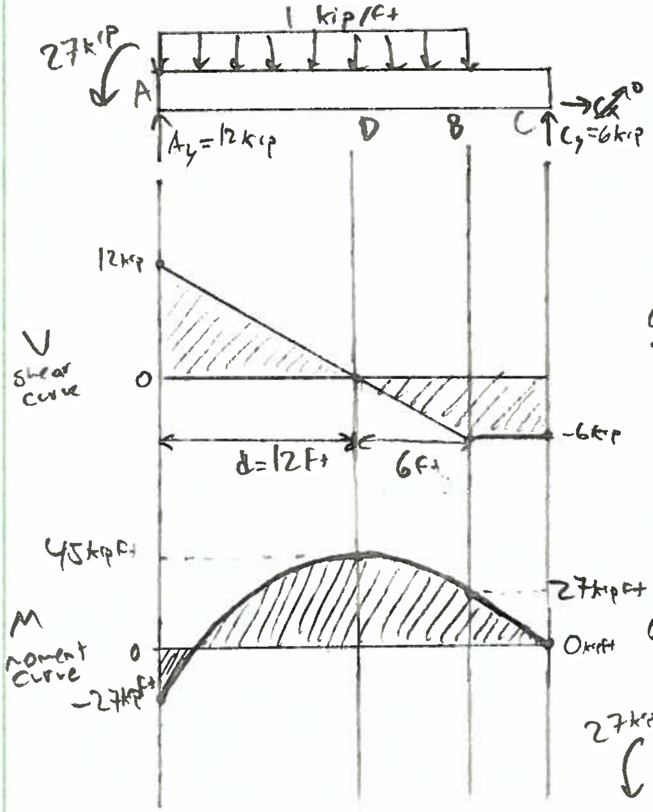
Note:
$\frac{dV}{dx} = -w \qquad V_D - V_C = -$(area under load curve btwn C & D)
$\frac{dM}{dx} = V \qquad M_D - M_C =$ (Area under shear curve btwn C & D)

$\circlearrowleft + \sum M_A = 0 \Rightarrow 27\,kip\,ft - (1\,kip/ft)(18\,ft)(9\,ft) + C_y(22.5\,ft) = 0$

$C_y = \frac{-27\,kip\,\cancel{ft} + (1\,kip/\cancel{ft})(18\,\cancel{ft})(9\,ft)}{22.5\,\cancel{ft}}$

$C_y = 6\,kip \uparrow$

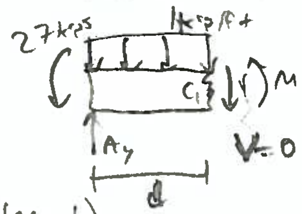
FBD of entire Beam



$+\uparrow \sum F_y = 0 \Rightarrow -(1\,kip/ft)(18\,ft) + C_y + A_y = 0$
$A_y = 18\,kip - 6\,kip$
$A_y = 12\,kip \uparrow$

Shear curve
$V_A = 12\,kips$
$V_B - V_A = -$[area under load curve A → B]
$V_B = V_A - [(1\,kip/ft)(18\,ft)]$
$V_B = 12\,kips - 18\,kips$, $V_B = -6\,kip$
$V_C - V_B = -$(area under load curve B → C)
$V_C = V_B + 0$, $V_C = -6\,kips$

At what distance from A is V = 0?
FBD of left side of cut C_1



$+\uparrow \sum F_y = 0 \Rightarrow A_y - (1\,kip/ft)d = 0$

$d = \frac{12\,\cancel{kip}}{1\,\cancel{kip}/ft}$, $d = 12\,ft$

Moment Curve
$M_A = -27\,kip\,ft$ (Due to external counterclockwise moment)

$M_D - M_A =$ (A under V curve A → D), $M_D = M_A + \frac{1}{2}(12\,kip)(12\,ft) = -27\,kip\,ft + 72\,kip\,ft$, $M_D = 45\,kip\,ft$

$M_B - M_D =$ (A under V curve D → B), $M_B = 45\,kip\,ft + \frac{1}{2}(-6\,kip)(6\,ft)$, $M_B = 27\,kip\,ft$

$M_C - M_B =$ (A under V curve B → C), $M_C = 27\,kip\,ft + (-6\,kip)(4.5\,ft)$, $M_C = 0\,kip\,ft$

Ch7
7.79 Part II)

From the graph we see that

$$\boxed{M_{max} = 45\,kipft \;@\; 12\,ft \text{ to the right of } A}$$

Our M_{max} occured where $V=0$

because of the relationship $\frac{dM}{dx} = V$,

There is a local max (or min depending on the case)

where $\frac{dM}{dx} = 0 = V$