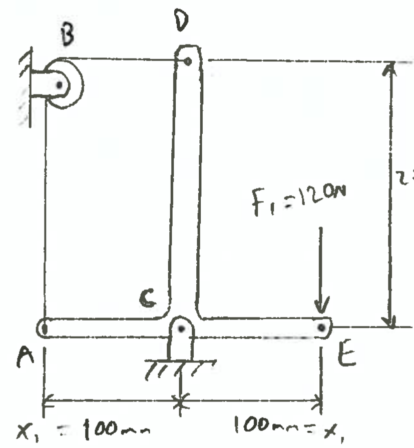
CH 4

VECTOR MECHANICS FOR ENGINEERS - STATICS - 9TH EDITION

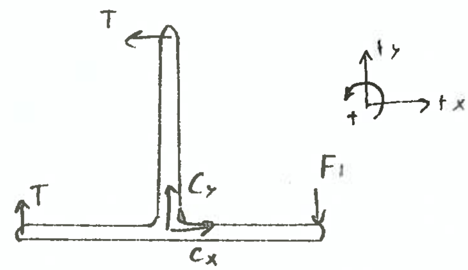
4.30) Pulley-member-pin System

Given: Neglect Friction

Find: T in cable ABD
Rxn at support C.



FBD of ACDE



$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \quad C_x - T = 0, \quad C_x = T \quad (1)$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow \quad T + C_y - F_1 = 0, \quad C_y = F_1 - T \quad (2)$$

$$\circlearrowleft^{+} \Sigma M_c = 0 \Rightarrow \quad y_1 T - x_1 T - x_1 F_1 = 0$$

$$T(y_1 - x_1) = x_1 F_1$$

$$T = \frac{x_1 F_1}{y_1 - x_1}, \quad T = \frac{(0.1m)(120N)}{(0.25m - 0.1m)}$$

$$\boxed{T = 80.0\,N}$$

(1)

$\Downarrow$

$C_x = 80.0N \rightarrow$

$$\boxed{\bar{C} = (80.0N, 40.0N)}$$

Or

$C = 89.4N \angle 26.6°$

(2)

$\Downarrow$

$$C_y = F_1 - \frac{x_1 F_1}{(y_1 - x_1)}$$

$$C_y = F_1\left(1 - \frac{x_1}{y_1 - x_1}\right)$$

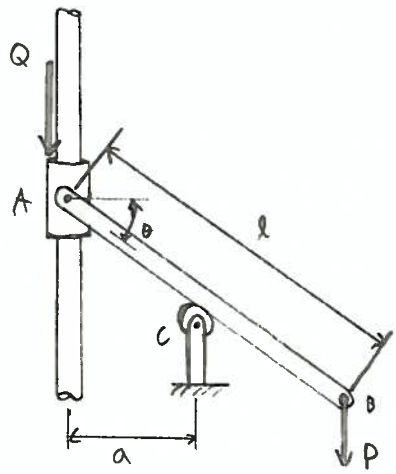
$$C_y = 120N\left[1 - \frac{(0.1m)}{(0.25m - 0.1m)}\right]$$

$C_y = 40.0\,N \uparrow$

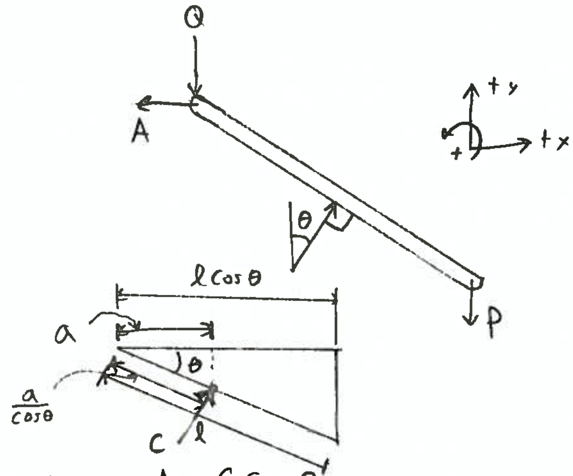
4.54) Rod - collar - roller system

Given: • Neglect weight of rod AB

Find: (a) Derive an equation in P, Q, a, l and θ that must be satisfied when the rod is in equilibrium
(b) Value of θ when P = 16 lb, Q = 12 lb, l = 20 in, and a = 5 in



FBD of rod AB (assuming no friction)



$+\rightarrow \Sigma F_x = 0 \Rightarrow C\sin\theta - A = 0 , \quad A = C\sin\theta$

$+\uparrow \Sigma F_y = 0 \Rightarrow C\cos\theta - Q - P = 0 , \quad C = \frac{Q+P}{\cos\theta}$

$\circlearrowleft + \ \Sigma M_A = 0 \Rightarrow C\left(\frac{a}{\cos\theta}\right) - P(l\cos\theta) = 0$

Now, $C = \frac{Q+P}{\cos\theta}$

So we have:

(a)
$$\left(\frac{Q+P}{\cos\theta}\right)\left(\frac{a}{\cos\theta}\right) - Pl\cos\theta = 0$$
$$\frac{(Q+P)a}{\cos^2\theta} = Pl\cos\theta , \quad \boxed{\cos^3\theta = \frac{(Q+P)a}{Pl}}$$

(b) Solving for θ:

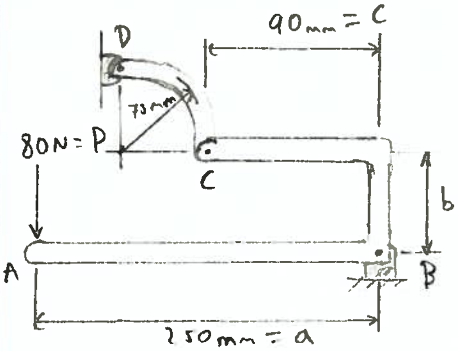
$$\cos\theta = \sqrt[3]{\frac{(Q+P)a}{Pl}} , \quad \theta = \cos^{-1}\left(\sqrt[3]{\frac{(Q+P)a}{Pl}}\right)$$

So, $\theta = \cos^{-1}\left(\sqrt[3]{\frac{(12\cancel{lb} + 16\cancel{lb})(5\cancel{in})}{(16\cancel{lb})(20\cancel{in})}}\right)$, $\boxed{\theta = 40.6^\circ}$

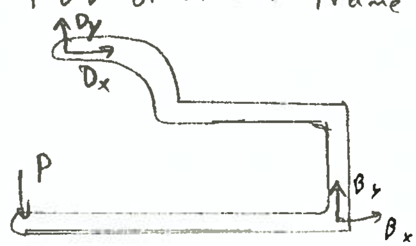
4.67) Two members - Two pin supports.

Given: $b = 120mm$

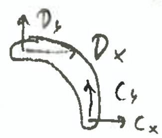
Find: Rxns at B and D.



FBD of entire frame

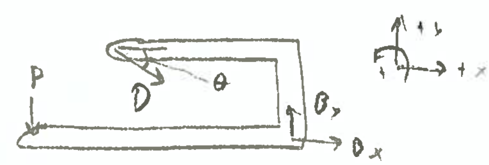


FBD of member DC



However since DC is a two force member we have:

FBD of member ABC



So, from FBD ABC:

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \quad B_x + D\cos\theta = 0, \quad B_x = -D\cos\theta$

$+\uparrow \Sigma F_y = 0 \Rightarrow \quad B_y - D\sin\theta - P = 0 \quad B_y = P + D\sin\theta$

$\circlearrowleft + \Sigma M_B = 0 \Rightarrow \quad aP + \begin{vmatrix} -c & b \\ D\cos\theta & -D\sin\theta \end{vmatrix} = 0$

$aP + [(-c)(-D\sin\theta) - (b)(D\cos\theta)] = 0$

$aP + Dc\sin\theta - Db\sin\theta = 0, \quad D(\cancel{+}c\sin\theta + b\sin\theta) = -aP$

$D = \frac{aP}{-c\sin\theta + b\sin\theta}, \quad D = \frac{(0.25m)(80N)}{-(0.09m)\sin 45° + (0.12m)\cos 45°}$

rxn @ D: $\boxed{D = 943N \quad 45°}$ or $\bar{D} = 943N(\cos 45°, -\sin 45°)$

So, $B_x = -(943N)\cos 45°$, $B_x = -667N$ $\quad B_y = 80N + (943N)\sin 45°$, $B_y = 747N$

So, $\bar{B} = (-667N, 747N)$

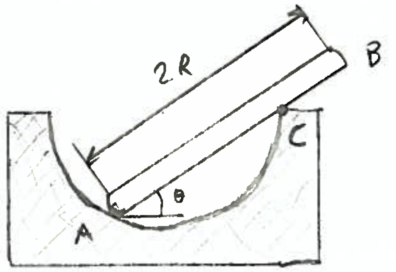
Or

$\boxed{B = 1001N \quad 48.2°}$

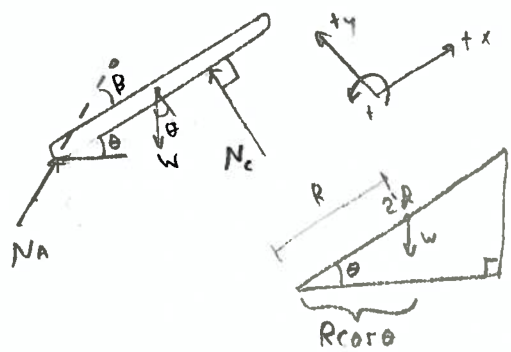
4.88) A rod resting inside a hemispherical bowl

Given: length of rod = 2R
radius of bowl = R
Neglect friction

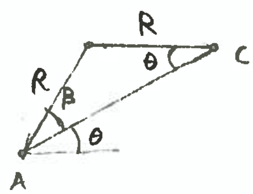
Find: θ corresponding to Equilibrium



FBD of rod AB



In order to find β
we have the following triangle:



Using law of Sines:

$$\frac{\cancel{R}}{\sin\beta} = \frac{\cancel{R}}{\sin\theta}, \quad \sin\beta = \sin\theta$$

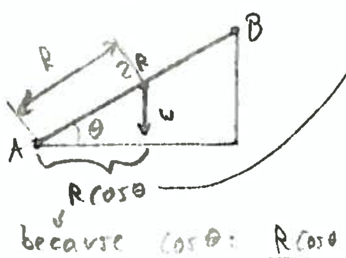
or

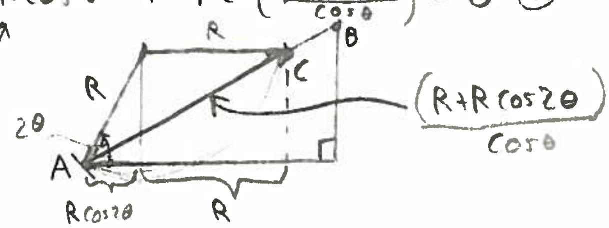
$$\beta = \theta$$

$$+\nearrow \Sigma F_x = 0 \Rightarrow \quad N_A\cos\theta - W\sin\theta = 0, \quad N_A = \frac{W\sin\theta}{\cos\theta} = W\tan\theta \quad (1)$$

$$+\nwarrow \Sigma F_y = 0 \Rightarrow \quad N_c - W\cos\theta + N_A\sin\theta = 0 \quad (2)$$

$$\circlearrowleft + \quad \Sigma M_A = 0 \Rightarrow \quad -W\cancel{R}\cos\theta + N_c\cancel{R}\left(\frac{1+\cos 2\theta}{\cos\theta}\right) = 0 \quad (3)$$





$$(3) \quad -W\cos\theta + N_c\left(\frac{1+\cos 2\theta}{\cos\theta}\right) = 0, \quad N_c\left(\frac{1+\cos 2\theta}{\cos\theta}\right) = W\cos\theta$$

$$N_c = \frac{W\cos^2\theta}{1+\cos 2\theta} \quad \text{but since } \cos 2\theta = 2\cos^2\theta - 1$$

$$N_c = \frac{W\cancel{\cos^2\theta}}{\cancel{1}+(2\cancel{\cos^2\theta}\cancel{-1})}, \quad N_c = \frac{W}{2} \quad (3)^*$$

4.86 Part II)

③ → ① → ②

$$\left(\frac{\cancel{W}}{2}\right) - \cancel{W}\cos\theta + \cancel{W}\tan\theta(\sin\theta) = 0$$

$$\frac{1}{2} - \cos\theta + \frac{\sin\theta}{\cos\theta}(\sin\theta) = 0$$

$$\frac{\cos\theta}{2} - \cos^2\theta + \sin^2\theta = 0$$

$$\frac{\cos\theta}{2} - \cos^2\theta + (1 - \cos^2\theta) = 0$$

$$-2\cos^2\theta + \frac{\cos\theta}{2} + 1 = 0$$

$$-4\cos^2\theta + \cos\theta + 2 = 0$$

Using quadratic equation:

$$\cos\theta = \frac{-1 \pm \sqrt{1^2 - (4)(-4)(2)}}{(-4)(2)}$$

$$\cos\theta_1 = \frac{-1 + \sqrt{33}}{-8}, \quad \theta_1 = 126.4°$$ → Does not make sense for our setup

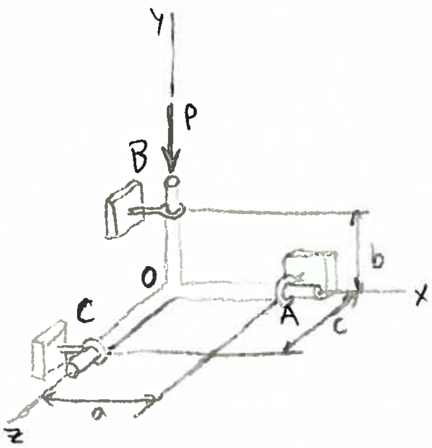
$$\cos\theta_2 = \frac{-1 - \sqrt{33}}{-8}, \quad \theta_2 = 32.5°$$ → makes sense

So for equilibrium $\boxed{\theta = 32.5°}$

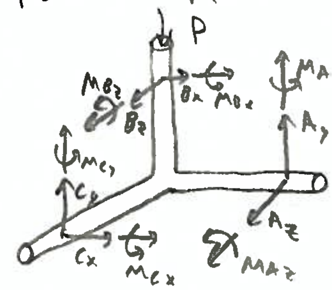
4.127) Three rods welded together, supported by three eyebolts.

Given: Neglect Friction
$P = 240\ lb$
$a = 12\ in$
$b = 8\ in$
$c = 10\ in$

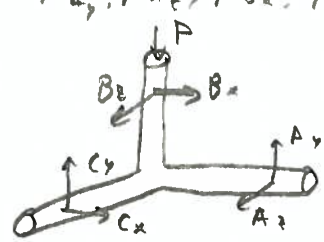
Find: Rxns @ A, B & C



FBD of ABC



However since we can form couples with the forces A_y, A_z, B_x, B_z, C_x & C_y that account for the reaction moments $M_{Ay}, M_{Az}, M_{Bx}, M_{Bz}, M_{Cx}$ & M_{Cy} we have:



which will give 6 eqns 6 unknowns.

$\Sigma F_x = 0 \Rightarrow C_x + B_x = 0 \quad , \quad C_x = -B_x$ (1)

$\Sigma F_y = 0 \Rightarrow C_y + A_y - P = 0 \, , \quad C_y = P - A_y$ (2)

$\Sigma F_z = 0 \Rightarrow A_z + B_z = 0 \quad , \quad A_z = -B_z$ (3)

$\Sigma \vec{M}_O = aA_y \hat{k} - aA_z \hat{j} - bB_x \hat{k} + bB_z \hat{i} + cC_x \hat{j} - cC_y \hat{i} = 0$

$\Sigma M_{Ox} = bB_z - cC_y = 0 \, , \quad B_z = \frac{cC_y}{b}$ (4)

$\Sigma M_{Oy} = cC_x - aA_z = 0 \, , \quad C_x = \frac{a}{c} A_z$ (5)

$\Sigma M_{Oz} = aA_y - bB_x = 0 \, , \quad A_y = \frac{bB_x}{a}$ (6)

(6) → (2)

$C_y = P - \frac{bB_x}{a}$

(1) → $C_y = P - \frac{b(-C_x)}{a} \, , \quad C_y = P + \frac{bC_x}{a}$

(5) → $C_y = P + \frac{b}{\cancel{a}}\left(\frac{\cancel{a}}{c} A_z\right) = P + \frac{b}{c} A_z$

(3) → $C_y = P + \frac{b}{c}(-B_z) = P - \frac{b}{c} B_z$

(4) → $C_y = P - \frac{\cancel{b}}{\cancel{c}}\left(\frac{\cancel{c}}{\cancel{b}} C_y\right)$

$C_y = \frac{P}{2} \quad , \quad C_y = \frac{240\ lb}{2}$

$C_y = 120\ lb$

4.127 Part II)

(2) $C_y = P - A_y$

$A_y = P - C_y$

$A_y = \frac{P}{2}$, $A_y = 120 lb$

(6) $A_y = \frac{b}{a} B_x$, $B_x = \frac{a}{b} A_y$, $B_x = \left(\frac{12 in}{8 in}\right)(120 lb)$

$B_x = 180 lb$

(1) $C_x = -B_x$, $C_x = -180 lb$

(5) $C_x = \frac{a}{c} A_z$, $A_z = \frac{c}{a} C_x$, $A_z = \left(\frac{10 in}{12 in}\right)(-180 lb)$

$A_z = -150 lb$

(3) $B_z = -A_z$, $\boxed{B_z = 150 lb}$

$$\boxed{\begin{aligned} \bar{A} &= (0,\ 120 lb,\ -150 lb) \\ \bar{B} &= (180 lb,\ 0,\ 150 lb) \\ \bar{C} &= (-180 lb,\ 120 lb,\ 0) \end{aligned}}$$