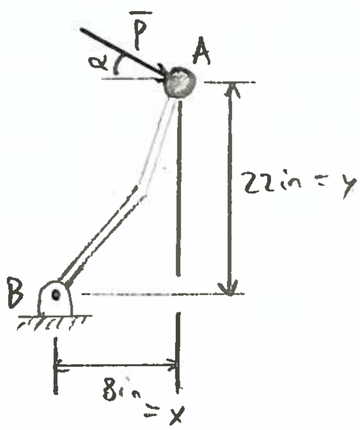
CH3

VECTOR MECHANICS FOR ENGINEERS - STATICS - 9TH EDITION

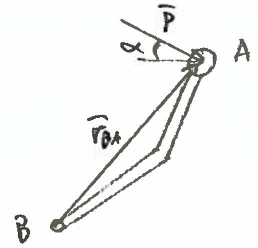
3.5) A Force applied to a shift lever.

Given: $P = 8\,lb$
$\alpha = 25°$

Find: M_B



FBD of Shift lever
(Not all Forces are included)



$\bar{P} = (P\cos\alpha, -P\sin\alpha)$

$\bar{r}_{BA} : (x, y)$

$\bar{M}_B = \bar{r}_{BA} \times \bar{P}$

Since 2D

$M_B = \begin{vmatrix} r_{BA_x} & r_{BA_y} \\ P_x & P_y \end{vmatrix} = \begin{vmatrix} x & y \\ P\cos\alpha & -P\sin\alpha \end{vmatrix}$

$M_B = x(-P\sin\alpha) - (y P\cos\alpha)$

$M_B = -P(x\sin\alpha + y\cos\alpha)$

$M_B = -(8\,lb)\left[(8in)\sin(25°) + (22in)\cos(25°)\right]$

$\boxed{M_B = -186.6\ lb\text{-}in}$

means clockwise moment

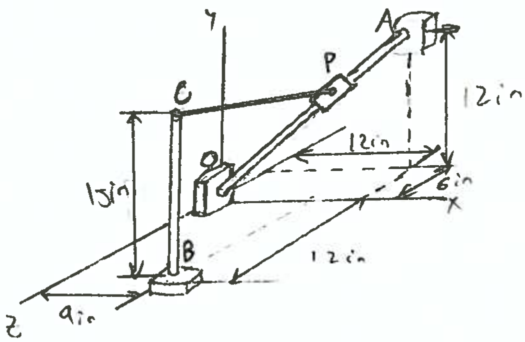
So we could write M_B as:

$M_B = 186.6\ lb\,in$ ↻

3.43) A collar - rod - cord system.

Given: · slider P can move along rod OA
· elastic cord PC is attached to slider and vertical member BC
· $|\overline{OP}| = 6in$
· $T = 3 lb$

Find: (a) The angle between the elastic cord and the rod OA
(b) the projection on OA of the force exerted by the cord PC at point P.



FBD of slider

In order to solve (a) and (b) we need the unit vectors $\frac{\overline{OA}}{|\overline{OA}|}$ and $\frac{\overline{PC}}{|\overline{PC}|} = \frac{\overline{T}}{T}$

$\overline{OA} : (12in, 12in, -6in)$

$$\frac{\overline{OA}}{|\overline{OA}|} = \frac{(12\cancel{in}, 12\cancel{in}, -6\cancel{in})}{\sqrt{(12\cancel{in})^2 + (12\cancel{in})^2 + (6\cancel{in})^2}} = \frac{1}{18}(12, 12, -6) = \widehat{OA}$$ unit vector in OA direction

Now, $\overline{OP} + \overline{PC} = \overline{OC}$, so, $\overline{PC} = \overline{OC} - \overline{OP}$

$$\overline{PC} = \overline{OC} - \underbrace{|\overline{OP}|}_{\text{mag of } \overline{OP}} \underbrace{\left(\frac{\overline{OA}}{|\overline{OA}|}\right)}_{\text{direction of } \overline{OP}}, \quad \overline{PC} = (9in, 13in, 12in) - \cancel{6in}\left(\frac{1}{\cancel{18}_3}(\cancel{12}, \cancel{12}, \cancel{-6})\right)$$ 4in 4in -2in

$\overline{PC} = (5in, 11in, 14in)$

$$\frac{\overline{PC}}{|\overline{PC}|} = \frac{(5in, 11in, 14in)}{\sqrt{(5in)^2 + (11in)^2 + (14in)^2}} = \frac{1}{3\sqrt{38}}(5, 11, 14) = \widehat{PC}$$ unit vector in PC direction.

a) Now $\widehat{OA} \cdot \widehat{PC} = \cancel{|\widehat{OA}|}\cancel{|\widehat{PC}|}\cos\theta$, where θ is the angle between PC and OA

unit vector mag = 1

So, $\cos\theta = \widehat{OA} \cdot \widehat{PC}$

3.43 Part II)

$$\cos\theta = \hat{OA} \cdot \hat{PC}$$

$$\cos\theta = \left[\frac{1}{18}(12, 12, -6)\right] \cdot \left[\frac{1}{3\sqrt{38}}(5, 11, 14)\right]$$

$$\cos\theta = \frac{1}{54\sqrt{38}}\left((12)(5) + (12)(11) + (-6)(14)\right)$$

$$\cos\theta = \frac{1}{54\sqrt{38}}(108)$$

$$\cos\theta = \frac{2}{\sqrt{38}} = \hat{OA} \cdot \hat{PC} \quad , \quad \theta = \cos^{-1}\left(\frac{2}{\sqrt{38}}\right)$$

$$\boxed{\theta = 71.1^\circ}$$

(b) Now, $\bar{T} = (T)\hat{PC}$

$\bar{T} =$

The projection of $\bar{T}$ onto $\bar{OA}$ is:

$$\bar{T} \cdot \hat{OA} = T \; \hat{PC} \cdot \hat{OA}$$

but since $\hat{PC} \cdot \hat{OA} = \hat{OA} \cdot \hat{PC} = \frac{2}{\sqrt{38}}$

we have

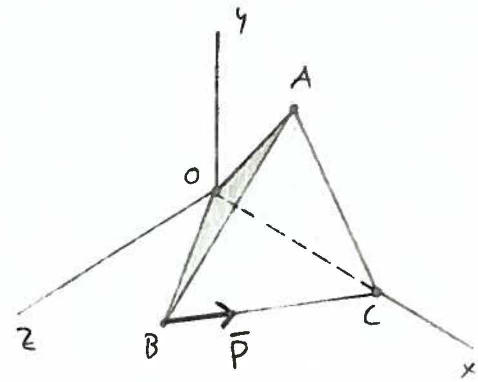
$$\bar{T} \cdot \hat{OA} = T\left(\frac{2}{\sqrt{38}}\right) = 3\,lb\left(\frac{2}{\sqrt{38}}\right)$$

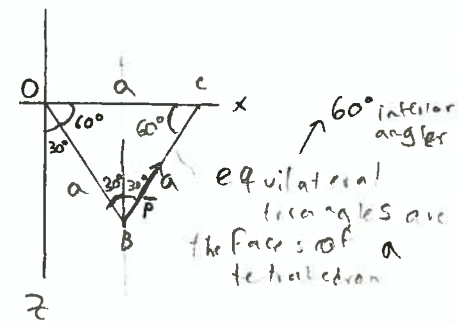
$$\boxed{\bar{T} \cdot \hat{OA} = 0.973\,lb}$$

3.59) A Force applied on a Tetrahedron

Given: length of each edge = a
$\bar{P}$ is directed along BC

Find: M_{OA} by P





So we have

$\bar{OB} = (a\cos 60°, 0, a\sin 60°)$

$\bar{P} = P\bar{\lambda}_{BC} = P\dfrac{(a\sin 30°, 0, -a\cos 30°)}{a}$

$\Rightarrow \bar{\lambda}_{OB} = (\cos 60°, 0, \sin 60°)$

Now, M_{OA} is given by:

$$M_{OA} = \underbrace{\bar{\lambda}_{OA}}_{\text{unit vector in direction of OA}} \cdot \bar{M}_O$$

where $\bar{M}_O = \bar{OB} \times \bar{P}$

$$\bar{M}_O = (a\cos 60°, 0, a\sin 60°) \times P(\sin 30°, 0, -\cos 30°)$$

$$\bar{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos 60° & 0 & a\sin 60° \\ P\sin 30° & 0 & -P\cos 30° \end{vmatrix} = \left[aP\sin 60°\sin 30° - (-aP\cos 60°\cos 30°)\right]\hat{j}$$

$$\bar{M}_O = \left[aP\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + aP\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right]\hat{j}$$

$$\bar{M}_O = \frac{aP\sqrt{3}}{2}\hat{j}$$

Now $\bar{\lambda}_{OA} \cdot \bar{\lambda}_{OB} = |\cancel{\bar{\lambda}_{OA}}|^{1}|\cancel{\bar{\lambda}_{OB}}|^{1}\cos 60°$, $\lambda_{OA_x}\cos 60° + \lambda_{OA_z}\sin 60° = \cos 60°$

and, $\underset{1\hat{i}}{\bar{\lambda}_{OC}} \cdot \bar{\lambda}_{OA} = |\cancel{\bar{\lambda}_{OC}}|^{1}|\cancel{\bar{\lambda}_{OA}}|^{1}\cos 60°$, $1\lambda_{OA_x} = \cos 60°$

$\lambda_{OA_x} = \cos 60°$

$(\cos 60°)(\cos 60°) + \lambda_{OA_z}\sin 60° = \cos 60°$

$$\lambda_{OA_z} = \frac{\cos 60° - \cos^2 60°}{\sin 60°}$$

3.59 Part II)

$\lambda_{OA_x} = \cos 60° = \frac{1}{2}$

$\lambda_{OA_z} = \frac{\cos 60° - \cos^2 60°}{\sin 60°} = \frac{\cos 60°}{\sin 60°}(1 - \cos 60°)$

we also know that $|\overline{OA}| = a$

$= \left(\frac{\cancel{\frac{1}{2}}}{\frac{\sqrt{3}}{\cancel{2}}}\right)\left(1 - \frac{1}{2}\right) = \frac{1}{2\sqrt{3}}$

So we have:

$$|\overline{\lambda_{OA}}| = 1 = \sqrt{(\lambda_{OA_x})^2 + (\lambda_{OA_y})^2 + (\lambda_{OA_z})^2}$$

Or

$$\lambda_{OA_y}^2 = 1 - \lambda_{OA_x}^2 - \lambda_{OA_z}^2$$

$$\lambda_{OA_y} = \sqrt{1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2\sqrt{3}}\right)^2}$$

$$\lambda_{OA_y} = \sqrt{1 - \frac{1}{4} - \frac{1}{12}}$$

$$\lambda_{OA_y} = \sqrt{\frac{2}{3}}$$

So, $M_{OA} = \overline{\lambda_{OA}} \cdot \overline{M_O}$

$$M_{OA} = \left(\frac{1}{2}, \sqrt{\frac{2}{3}}, \frac{1}{2\sqrt{3}}\right) \cdot \left(0, \frac{aP\sqrt{3}}{2}, 0\right)$$

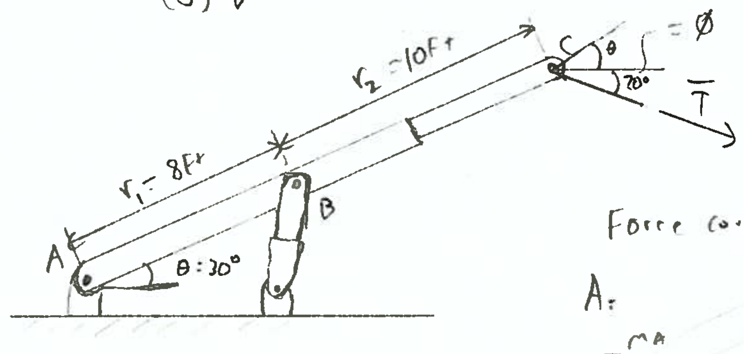
$$M_{OA} = \left(\frac{\sqrt{2}}{\cancel{\sqrt{3}}}\right)\left(\frac{aP\cancel{\sqrt{3}}}{\underbrace{2}_{(\sqrt{2})(\sqrt{2})}}\right) \quad \boxed{M_{OA} = \frac{aP}{\sqrt{2}}}$$

3.81) tension on a cable attached to an adjustable boom.

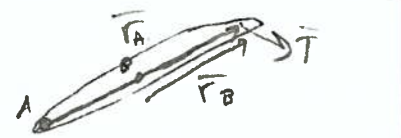
Given: T: 560 lb

Find: Equivalent Force couple system at

(a) A
(b) B

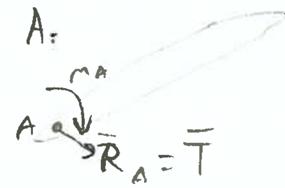


FBD of AC (Not all Forces shown)

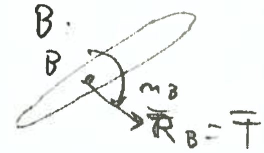


Force couple system @

A:



B:



$\bar{M}_A = \bar{r}_A \times \bar{T}$

↓ 2D

$M_A = \ominus r_A T \sin(\theta + \phi)$

by right hand rule

$M_A = -(r_1 + r_2)\, T \sin(\theta + \phi)$

$M_A = -(8ft + 10ft)(560 lb) \sin(20° + 30°)$

$$\boxed{\begin{array}{l} M_A = 7720 \text{ ft lb} \curvearrowright \\ \bar{R}_A = 560 \text{ lb} \searrow 20° \end{array}}$$

$M_B = \bar{r}_B \times \bar{T}$

↓ 2D

$M_B = \ominus r_B T \sin(\theta + \phi)$

by right hand rule

$M_B = -(r_2)\, T \sin(\theta + \phi)$

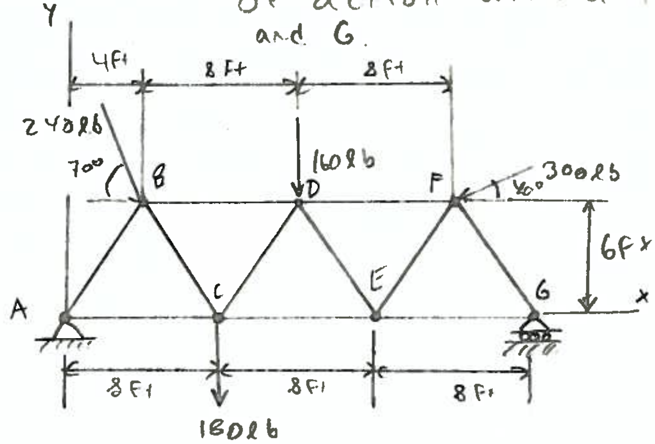
$M_B = -(10ft)(560 lb) \sin(20° + 30°)$

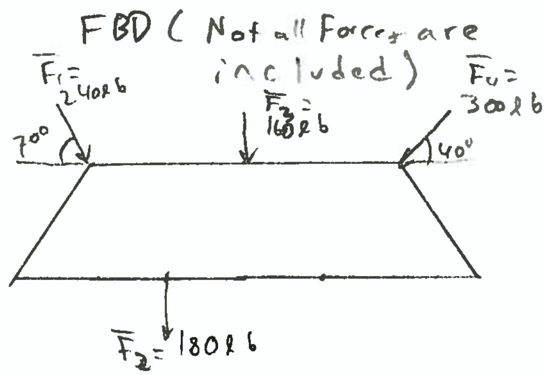
$$\boxed{\begin{array}{l} M_B = 4290 \text{ ft lb} \curvearrowright \\ \bar{R}_B = 560 \text{ lb} \searrow 20° \end{array}}$$

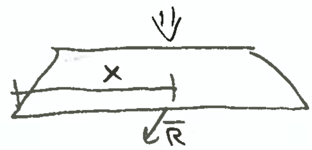
3.113) loads on a truss.

Given: The Following Space diagram

Find: The equivalent Force acting on the truss and the point of intersection of its line of action with a line drawn through points A and G.







$$\Sigma \bar{F} = \bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$\bar{R} = (F_1 \cos 70°, -F_1 \sin 70°) + (0, -F_2) + (0, -F_3) + (-F_4 \cos 40°, -F_4 \sin 40°)$$

$$\bar{R} = (F_1 \cos 70° - F_4 \cos 40°, -F_1 \sin 70° - F_2 - F_3 - F_4 \sin 40°)$$

$$\bar{R} = ((240 lb)\cos 70° - (300 lb)\cos 40°, -(240 lb)\sin 70° - 180 lb - 160 lb - (300 lb)\sin 40°)$$

$$\boxed{\bar{R} = (-147.7 lb, -758 lb)} \Rightarrow \text{or} \Rightarrow \boxed{R = 773 lb \quad \alpha = 79.0°}$$

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-758.363}{-147.7}\right)$$

$$\Sigma M_A = \overline{AB} \times \bar{F}_1 + \overline{AC} \times \bar{F}_2 + \overline{AD} \times \bar{F}_3 + \overline{AF} \times \bar{F}_4$$

$$\Sigma M_A = -(6ft)(240lb)\cos 70° - (4ft)(240lb)\sin 70° - (8ft)(180lb) - (12ft)(160lb) - (20ft)(300lb)\sin 40° + (6ft)(300lb)\cos 40°$$

$$\Sigma M_A = -7232.4595 lb ft$$

We can remove the M_A if we move $\bar{R}$ to the right a distance x,

Such that: $\Sigma M_A = R_y x$

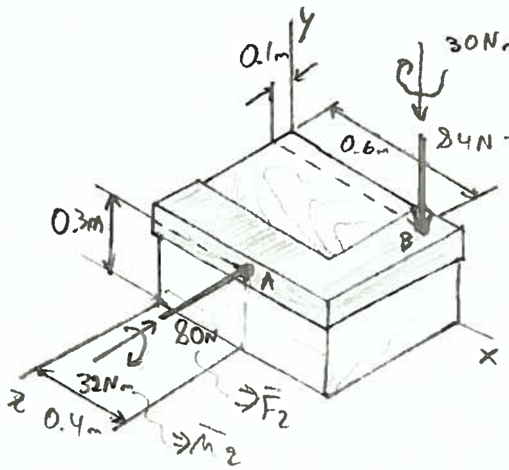
So, $x = \frac{\Sigma M_A}{R_y}$, $x = \frac{-7232.4595 lb ft}{-758.363 lb}$

$\boxed{x = 9.54 ft}$ Or 9.54 ft to the right of A.

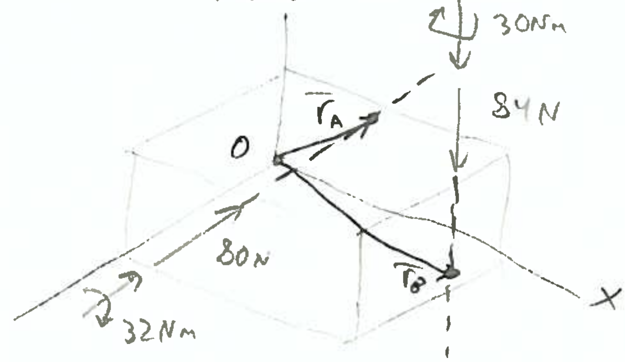
3.137) Two bolts being tightened

Given: The following space diagram

Find: (a) The resultant $\bar{R}$
(b) the pitch of the single equivalent wrench
(c) The point where the axis of the wrench intersects the x z plane.



FBD



(a) $\bar{R} = \bar{F}_1 + \bar{F}_2 = -F_1\hat{j} - F_2\hat{k} = \boxed{-84.0N\hat{j} - 80.0N\hat{k} = \bar{R}}$

$$\Sigma \bar{M}_O = \bar{M}_1 + \bar{M}_2 + \bar{r}_B \times \bar{F}_1 + \bar{r}_A \times \bar{F}_2$$

O is the Origin.

$$\Sigma \bar{M}_O = \bar{M}_1 + \bar{M}_2 + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.4m & 0.3m & 0 \\ 0 & 0 & -80N \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6m & 0 & 0.1m \\ 0 & -84N & 0 \end{vmatrix}$$

$$\Sigma \bar{M}_O = -30Nm\,\hat{j} - 32Nm\,\hat{k} + (0.3m)(-80N)\hat{i} - (0.4m)(-80N)\hat{j} - (0.1m)(-84N)\hat{i} + (0.6m)(-84N)\hat{k}$$

$$\Sigma \bar{M}_O = -15.6Nm\,\hat{i} + 2Nm\,\hat{j} - 82.4Nm\,\hat{k}$$

So the pitch is:

$$p = \frac{\bar{R}\cdot\bar{M}_O^R}{R^2} = \frac{(-84N)(2Nm) + (-80N)(-82.4Nm)}{\left(\sqrt{(-84N)^2 + (-80N)^2}\right)^2} = \frac{6424\,N^2m}{13456\,N^2} = \frac{803}{1682}\,m$$

(b) $\boxed{p = 0.477m}$

3.137 Part II)

(c) in order to find the point where the axis of the wrench intersects the xz plane we will use the equation:

$$\rho\bar{R} + \bar{r}\times\bar{R} = \bar{M}_o^R = \Sigma\bar{M}_o$$

Coordinate of intersection:

$$(r_x, 0, r_z)$$

$$\rho\bar{R} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & 0 & r_z \\ 0 & R_y & R_z \end{vmatrix} = \Sigma\bar{M}_o$$

$$(0 - r_z R_y)\hat{i} + (0 - r_x R_z)\hat{j} + r_x R_y = \Sigma\bar{M}_o - \rho\bar{R}$$

x: $$-r_z R_y = \Sigma M_{ox} - \cancelto{0}{\rho R_x}$$

$$r_z = -\frac{\Sigma M_{ox}}{R_y}, \quad r_z = \frac{-(+15.6\,Nm)}{+84N} = -0.1857m$$

z: $$r_x R_y = \Sigma M_{oz} - \rho R_z$$

$$r_x = \frac{\Sigma M_{oz} - \rho R_z}{R_y}$$

$$r_x = \frac{-82.4\,Nm + \left(\frac{803}{1682}m\right)(+80N)}{-84N}$$

$$r_x = 0.526m$$

So the point where the axis of the wrench intersects the xz plane is

$$\boxed{(0.526m,\ 0,\ -0.1857m)}$$