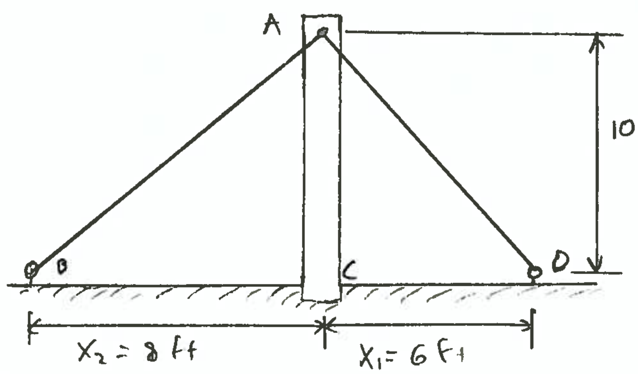
CH2

VECTOR MECHANICS FOR ENGINEERS - STATICS - 9TH EDITION

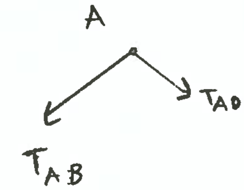
2.3) - Cables supporting a pole.

Given: $T_{AB} = 120\,lb$
$T_{AD} = 40\,lb$

Find: $\bar{R}$ graphically using: (a) Parallelogram law
(b) triangle rule

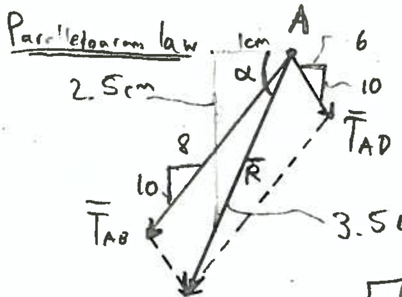


FBD, point A (Only tensions)



Finding $\bar{R}$ graphically:

Parallelogram law

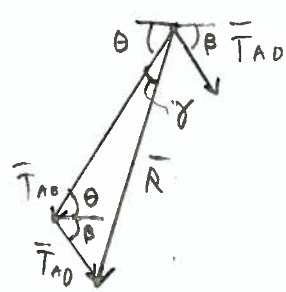


3.5 cm ⇒ $R = 3.5\,cm \cdot \frac{40\,lb}{1\,cm}$

(1 cm = 40 lb)

$$\boxed{R = 140\,lb \qquad \alpha = \tan^{-1}\left(\frac{2.5}{1}\right) \qquad \alpha = 68.2^\circ}$$

triangle rule



Using Law of cosines:

$$R^2 = T_{AB}^2 + T_{AD}^2 - 2T_{AB}T_{AD}\cos(\theta+\beta)$$

$$R = \sqrt{T_{AB}^2 + T_{AD}^2 - 2T_{AB}T_{AD}\cos(\theta+\beta)}$$

Now, $\theta = \tan^{-1}\left(\frac{y_1}{x_2}\right)$
$\beta = \tan^{-1}\left(\frac{y_1}{x_1}\right)$

So, $R = \sqrt{T_{AB} + T_{AD} - 2T_{AB}T_{AD}\cos\left(\tan^{-1}\left(\frac{y_1}{x_2}\right) + \tan^{-1}\left(\frac{y_1}{x_1}\right)\right)}$

$R = \sqrt{(120\,lb)^2 + (40\,lb)^2 - 2(120\,lb)(40\,lb)\cos\left(\tan^{-1}\left(\frac{10}{8}\right) + \tan^{-1}\left(\frac{10}{6}\right)\right)}$

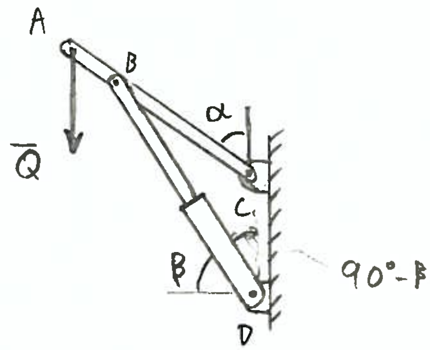
$\boxed{R = 139.1\,lb}$ ⇒ angle ⇒ $\theta + \gamma = \tan^{-1}\left(\frac{10}{8}\right) + 15.64^\circ = \boxed{67.0^\circ}$

Finding γ: Law of sines: $\frac{R}{\sin(\theta+\beta)} = \frac{T_{AD}}{\sin\gamma}$, $\gamma = \sin^{-1}\left(\frac{\sin(\theta+\beta)\,T_{AD}}{R}\right) \Rightarrow \gamma = 15.64^\circ$

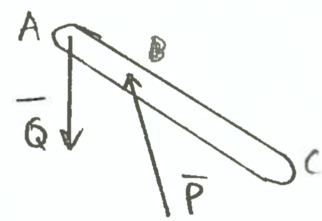
2.26) Hydraulic Cylinder applying a Force on a member.

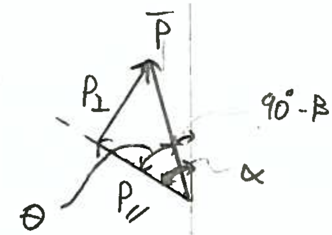
Given: $\alpha = 60°$
$\beta = 50°$
P is directed along BD
P has a 750N component $\perp$ to ABC ($P_\perp = 750N$

Find: (a) P
(b) P's Component // to ABC



FBD of ABC (Not all F's are included)





So, as seen here, $\theta = \alpha - (90° - \beta) = \alpha + \beta - 90°$

And, $\sin\theta = \dfrac{P_\perp}{P}$, $P = \dfrac{P_\perp}{\sin\theta}$

$$P = \frac{P_\perp}{\sin(\alpha + \beta - 90°)} = \frac{750N}{\sin(60° + 50° - 90°)}, \quad \boxed{P = 2190N}$$

Also, $\tan\theta = \dfrac{P_\perp}{P_{//}}$, $P_{//} = \dfrac{P_\perp}{\tan\theta}$, $P_{//} = \dfrac{P_\perp}{\tan(\alpha + \beta - 90°)}$

$$P_{//} = \frac{750N}{\tan(60° + 50° - 90°)}, \quad \boxed{P_{//} = 2060N}$$

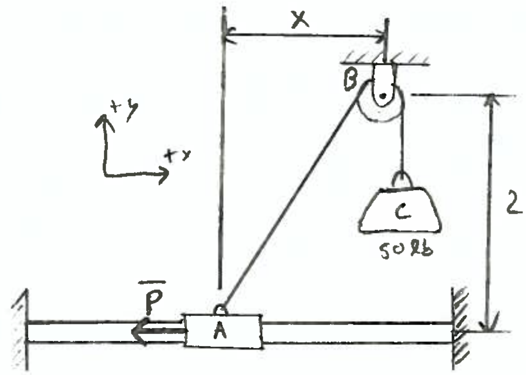
2.63) Equilibrium of a Collar-Pulley-mass system.

Given: $W_c = 50\,lb$

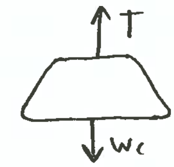
Collar A can slide on the Horizontal <u>frictionless rod</u>

Find: P in order to maintain equib. if (a) $x = 4.5\,in$

(b) $x = 15\,in$



FBD of C



$+\uparrow \Sigma F_y = 0$

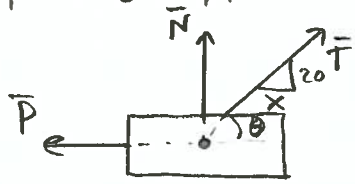
$\Downarrow$

$T - W_c = 0$

$T = W_c$

Since the system is in equib, $\Sigma \bar{F} = 0$

FBD of A



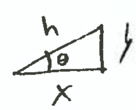
$\rightarrow \Sigma F_x = 0 \Rightarrow T\cos\theta - P = 0, \quad P = T\cos\theta$

$+\uparrow \Sigma F_y = 0 \Rightarrow T\sin\theta + N = 0$

(Because the rod is frictionless there is no force due to the rod on the collar parallel to the rod)

Since $T = W_c$

$P = W_c \cos\theta$



$h = \sqrt{y^2 + x^2}$

$$\cos\theta = \frac{x}{h} = \frac{x}{\sqrt{y^2+x^2}}$$

So, $$P = W_c\left(\frac{x}{\sqrt{y^2+x^2}}\right)$$

(a) $x = 4.5\,in$

$$P = 50\,lb\left(\frac{4.5\,in}{\sqrt{(20\,in)^2 + (4.5\,in)^2}}\right), \quad \boxed{P = 10.98\,lb}$$

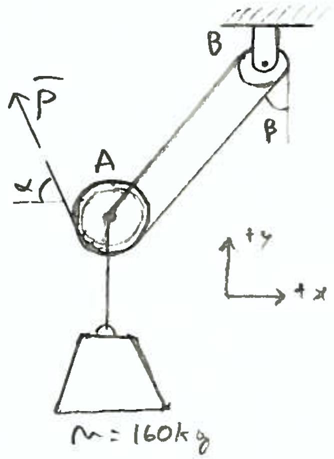
(b) $x = 15\,in$

$$P = 50\,lb\left(\frac{15\,in}{\sqrt{(20\,in)^2 + (15\,in)^2}}\right), \quad \boxed{P = 30.0\,lb}$$

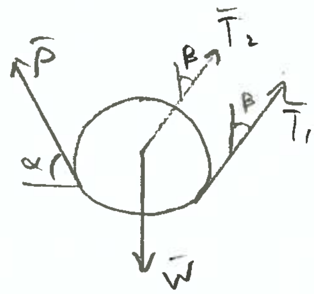
2.65) Rope and Pulley arrangement

Given: $M = 160\,kg$
$\beta = 20°$

Find: $\bar{P}$ required to maintain equib



FBD of A



Where,
$T_1 = T_2 = P$
(Because Pulley is frictionless)

$$+\rightarrow \Sigma F_x = \underset{P}{T_1}\sin\beta + \underset{P}{T_2}\sin\beta - P\cos\alpha = 0$$

$$\cancel{P}\sin\beta + \cancel{P}\sin\beta - \cancel{P}\cos\alpha = 0$$

$$2\sin\beta = \cos\alpha, \quad \alpha = \cos^{-1}(2\sin\beta)$$

$$\alpha = \cos^{-1}(2\sin(20°)), \quad \alpha = 46.8°$$

$$+\uparrow \Sigma F_y = T_1\cos\beta + T_2\cos\beta + P\sin\alpha - W = 0$$

$$2P\cos\beta + P\sin\alpha - mg = 0$$

$$P(2\cos\beta + \sin\alpha) = mg$$

$$P = \frac{mg}{2\cos\beta + \sin(\cos^{-1}(2\sin\beta))}$$

$$P = \frac{(160\,kg)(9.81\,m/s^2)}{2\cos 20° + \sin(46.84°)}$$

$\bar{P} \Rightarrow$ $\boxed{P = 602\,N \;\; \alpha\measuredangle, \; \alpha = 46.8°}$ or $\bar{P} = 602N(\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j})$

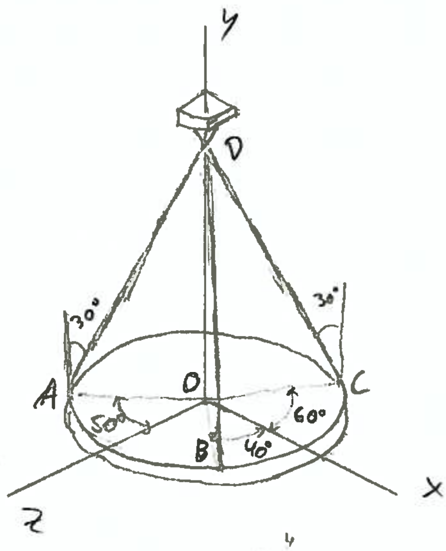
2.73) A horizontal circular plate suspended from 3 wires.

Given: • The 3 wires are attached to support D and form 30° angles with the vertical.

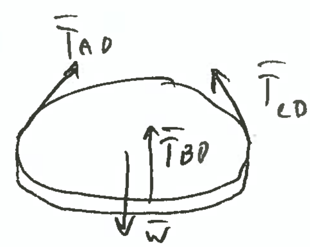
• $T_{AD_x} = 110.3\ N$

Find: (a) T_{AD}

(b) θ_x, θ_y & θ_z of $\bar{T}_{BD}$

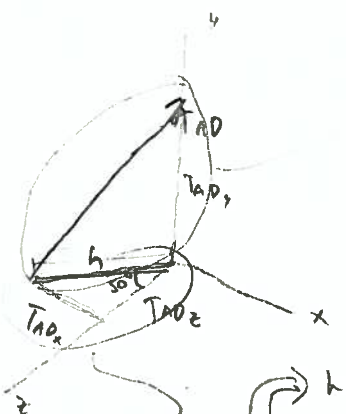


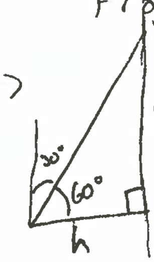
FBD of Plate



(a) we need to find T_{AD}

from: $\bar{T}_{AD} = T_{ADx}\hat{i} + T_{ADy}\hat{j} + T_{ADz}\hat{k}$





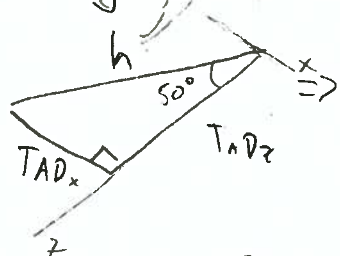
T_{AD_x} is known, so, we need to find T_{AD_y} and T_{AD_z}

$$\tan 60° = \frac{T_{AD_y}}{h}$$

$$T_{AD_y} = h \tan 60°$$

$$T_{AD_y} = \sqrt{T_{AD_x}^2 + T_{AD_z}^2}\ \tan 60° \quad (*)$$

$$h = \sqrt{T_{AD_x}^2 + T_{AD_z}^2}$$



$$\tan 50° = \frac{T_{ADx}}{T_{ADz}}, \quad T_{ADz} = \frac{T_{AD_x}}{\tan 50°}$$

$$T_{ADz} = \frac{110.3N}{\tan 50°} = 92.55$$

So from (*)

$$T_{AD_y} = \sqrt{(110.3N)^2 + (92.55N)^2}\ \tan 60°, \quad T_{AD_y} = 249.39N$$

And, $T_{AD} = \sqrt{T_{AD_x}^2 + T_{AD_y}^2 + T_{AD_z}^2}$, $T_{AD} = \sqrt{(110.3N)^2 + (249.39N)^2 + (92.55N)^2}$

$$\boxed{T_{AD} = 288N}$$

2.73 Part II)

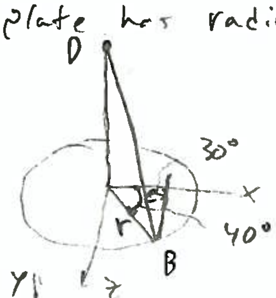
(b) In order to find θ_x, θ_y, θ_z for T_{BD} we need to use the relationship:

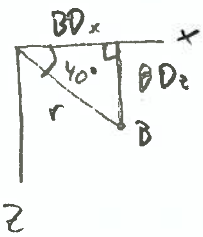
$$\frac{\overline{T_{BD}}}{|T_{BD}|} = \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$$

However since $\frac{\overline{T_{BD}}}{T_{BD}}$ is simply the unit vector for T_{BD}

We can also express it as: $\frac{\overline{BD}}{|\overline{BD}|} = \frac{BD_x \hat{i} + BD_y \hat{j} + BD_z \hat{k}}{\sqrt{BD_x^2 + BD_y^2 + BD_z^2}}$

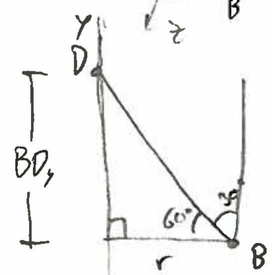
The plate has radius r





$\sin 40° = \frac{BD_z}{r}$, $BD_z = r\sin 40°$

$\cos 40° = \frac{BD_x}{r}$, $BD_x = r\cos 40°$



$\tan 60° = \frac{BD_y}{r}$, $BD_y = r\tan 60°$

So $$\frac{\overline{BD}}{|\overline{BD}|} = \frac{r\cos 40° \hat{i} + r\tan 60° \hat{j} + r\sin 40° \hat{k}}{\sqrt{(r\cos 40°)^2 + (r\tan 60°)^2 + (r\sin 40°)^2}}$$

$$= \frac{\cancel{r}\,(\cos 40° \hat{i} + \tan 60° \hat{j} + \sin 40° \hat{k})}{\cancel{r}\sqrt{\underbrace{\cancel{\cos^2 40° + \sin^2 40°}}_{1} + \tan^2 60°}}$$

$$1 + \tan^2 60° = 1 + \frac{\sin^2 60°}{\cos^2 60°} = \frac{\cancel{\cos^2 60° + \sin^2 60°}\ 1}{\cos^2 60°} = \frac{1}{\cos^2 60°}$$

$$= \frac{\cos 40° \hat{i} + \tan 60° \hat{j} + \sin 40° \hat{k}}{\sqrt{\frac{1}{\cos^2 60°}}}$$

$$\frac{\overline{BD}}{|\overline{BD}|} = \cos 60° (\cos 40° \hat{i} + \tan 60° \hat{j} + \sin 40° \hat{k}) = \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$$

2.73 Part III)

x direction: $(\cos 60^\circ)(\cos 40^\circ) = \cos \theta_x$

$$\theta_x = \cos^{-1}\left(\cos 60^\circ \cos 40^\circ\right), \quad \boxed{\theta_x = 67.5^\circ}$$

y: $\cancel{\cos 60^\circ}\ \cancel{\tan 60^\circ}$ $\sin 60^\circ = \cos \theta_y$

$$\theta_y = \cos^{-1}\left(\sin 60^\circ\right)$$

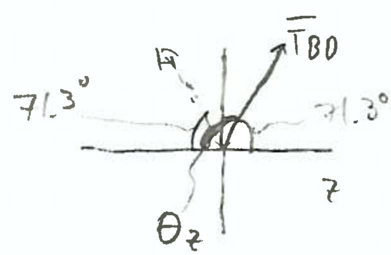
$$\boxed{\theta_y = 30^\circ}$$

z: $(\cos 60^\circ)(\sin 40^\circ) = \cos \theta_z$

$$\theta_z = \underbrace{\cos^{-1}\left(\cos 60^\circ \sin 40^\circ\right)}_{71.3}$$

However, the angle is $> 90^\circ$

$\Downarrow$



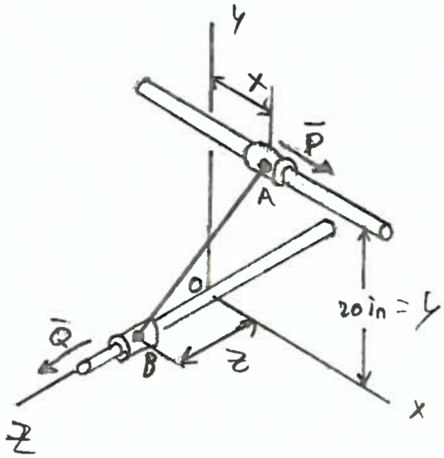
So, $\theta_z = 180 - 71.3^\circ$

$$\boxed{\theta_z = 108.7^\circ}$$

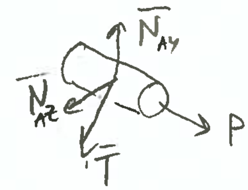
2.126) Rod - Collar - Cable System

Given: · wire is 25 in long. ($L = 25$ in)
· rods are frictionless
· $P = 120$ lb
· $Q = 60$ lb

Find: x & z to maintain equilibrium



FBD of A



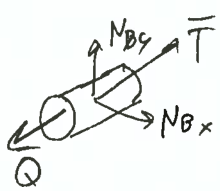
$\Sigma F_x = 0 \Rightarrow P - T_x = 0$
$T_x = P$, $T_x = 120$ lb

$\Sigma F_y = 0 \Rightarrow N_{Ay} - T_y = 0$, $T_y = N_y$

$\Sigma F_z = 0 \Rightarrow N_{Az} - T_z = 0$, $T_z = N_z$

(Note opposite sign, equal and opposite force)

FBD of B



$\Sigma F_x = 0 \Rightarrow (+) T_x + N_{Bx} = 0$, $T = -N_{Bx}$

$\Sigma F_y = 0 \Rightarrow (+) T_y + N_{By} = 0$, $T_y = -N_{By}$

$\Sigma F_z = 0 \Rightarrow (+) T_z + Q = 0$, $T_z = -Q$
$T_z = -60$ lb

Now, $\bar{T}$ in FBD B $\Rightarrow \bar{T} = (T_x, T_y, T_z) = (120, T_y, -60)$ lb

And, $\dfrac{\overline{BA}}{|BA|} = \dfrac{(x, y, -z)}{L} = \dfrac{(T_x, T_y, T_z)}{\sqrt{T_x^2 + T_y^2 + T_z^2}} = \dfrac{\bar{T}}{T}$

unit vector of $\bar{T}$ $\quad L = \sqrt{x^2 + y^2 + z^2}$

So, we have:

x: $\dfrac{x}{L} = \dfrac{T_x}{\sqrt{T_x^2 + T_y^2 + T_z^2}}$ (1)

y: $\dfrac{y}{L} = \dfrac{T_y}{\sqrt{T_x^2 + T_y^2 + T_z^2}}$ (2)

z: $\dfrac{-z}{L} = \dfrac{T_z}{\sqrt{T_x^2 + T_y^2 + T_z^2}}$ (3)

3 eqns
Knowns: T_x, T_z, y, L
Unknowns: T_y, x, z

2.126 Part II)

(2) $\Rightarrow$

$$\frac{y}{L} = \frac{T_x}{\sqrt{T_x^2 + T_y^2 + T_z^2}}$$

$$\frac{y^2}{L^2} = \frac{T_y^2}{T_x^2 + T_y^2 + T_z^2}$$

Solving for T_y:

$$\frac{y^2}{L^2}\left(T_x^2 + T_y^2 + T_z^2\right) = T_y^2$$

$$T_x^2 + T_y^2 + T_z^2 = \frac{T_y^2 L^2}{y^2}$$

$$\frac{T_y^2 L^2}{y^2} - T_y^2 = T_x^2 + T_z^2$$

$$T_y^2 = \frac{T_x^2 + T_z^2}{\frac{L^2}{y^2} - 1} \qquad (2)^*$$

$(2)^* \Rightarrow (1)$

$$\frac{y}{L} = \frac{T_x}{\sqrt{T_x^2 + \frac{T_x^2 + T_z^2}{\left(\frac{L^2}{y^2} - 1\right)} + T_z^2}}$$

$$\frac{x}{L} = \frac{T_x}{\sqrt{\frac{T_x^2\left(\frac{L^2}{y^2} \cancel{-1}\right) \cancel{+ T_x^2} + \cancel{T_z^2} + T_z^2\left(\frac{L^2}{y^2} \cancel{-1}\right)}{\left(\frac{L^2}{y^2} - 1\right)} \cdot \frac{\left(\frac{y^2}{L^2}\right)}{\left(\frac{y^2}{L^2}\right)}}}$$

$$X = \frac{T_x L}{\sqrt{\frac{T_x^2 + T_z^2}{1 - \frac{y^2}{L^2}}}}, \quad \text{where } T_x = P, \; T_z = -Q$$

$$x = \frac{PL}{\sqrt{\frac{P^2 + Q^2}{1 - \frac{y^2}{L^2}}}}, \quad x = \frac{(120\,lb)(25\,in)}{\sqrt{\frac{(120\,lb)^2 + (60\,lb)^2}{1 - \frac{(20\,in)^2}{(25\,in)^2}}}}$$

$$\boxed{x = 13.42 \text{ in}}$$

2.126 Part III)

(2)* $\Rightarrow$ (3)

$$\frac{z}{L} = \frac{T_z}{\sqrt{T_x^2 + \dfrac{T_x^2 + T_z^2}{\left(\frac{L^2}{y^2} - 1\right)} + T_z^2}}$$

Very similar to when we solved for x,

So we have:

$$z = \frac{T_z L}{\sqrt{\dfrac{T_x^2 + T_z^2}{1 - \frac{y^2}{L^2}}}} = \frac{QL}{\sqrt{\dfrac{P^2 + Q^2}{1 - \frac{y^2}{L^2}}}}, \quad z = \frac{(60\,lb)(25\,in)}{\sqrt{\dfrac{(20\,lb)^2 + (60\,lb)^2}{1 - \frac{(10\,in)^2}{(25\,in)^2}}}}$$

$$\boxed{z = 6.71\,in}$$